PHYS 705: Classical Mechanics

Pre-class notes:

- Double Pendulum demo
- Catch up week: you will have two weeks to do HW#5 (due Oct 12)
- Due to Columbus Day Holiday, class will meet on Tuesday (Oct 12) instead of Monday (Oct 11)

HW#3:

Newton’s 2nd Law:

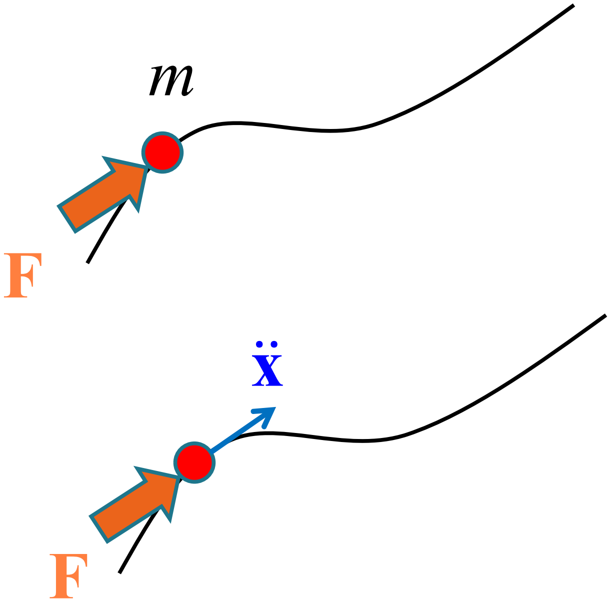
- Mass m is being pushed by a net force $\mathbf{F}$

- Its motion (its acceleration) $\ddot{\mathbf{x}}$ is given by the 2nd law

$$\frac{d\mathbf{P}}{dt} = m\ddot{\mathbf{x}} = \mathbf{F}$$

how it moves

the push



$m\ddot{\mathbf{x}}$ is NOT a force !

Conservation and Symmetry

There is a very strong link between Symmetry and Conservation Theorems:

- Symmetry ⇔ conserved quantity
- New Terminology:

$$\left.\begin{array}{c}\text{generalized momentum}\\ \text{canonical momentum}\\ \text{conjugate momentum}\end{array}\right\} \equiv \boxed{p_j \equiv \frac{\partial L}{\partial \dot{q}_j}}$$

cyclic coordinate $\equiv$ one that doesn't appear in L, $i.e., \dfrac{\partial L}{\partial q_j} = 0$

Conservation and Symmetry

Let consider a system in which all applied forces are derivable from a scalar potential function and we have chosen a set of "proper" generalized coordinates (i.e., all constraints are "hidden"). Then,

$$\frac{\partial L}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = 0$$

A generalized coordinate q_j is **cyclic** if $\frac{\partial L}{\partial q_j} = 0$

Then, EL equation implies $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = 0$

⇒ $p_j \equiv \frac{\partial L}{\partial \dot{q}_j} = const$ p_j is conserved !

Conservation and Symmetry

Simple examples:

→ If q_j corresponds to a translation relative to fixed Cartesian coordinates

⇒ Conservation of Linear Momentum

→ If q_j corresponds to a rotation of the system

⇒ Conservation of Angular Momentum

→ In general, if q_j is a cyclic variable, its corresponding conjugate momentum p_j will be conserved.

Energy Conservation and Time Invariance

Defining $h = \sum_j \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L$

1. If t is a cyclic variable (L is time invariance), h is conserved.

$\frac{\partial L}{\partial t} = 0$ ⇒ $\frac{dh}{dt} = -\frac{\partial L}{\partial t} = 0$ h is constant in time!

2. Additionally, if

$U = U(q_i)$ (U does not dep on $\dot{q}_i$) AND

$\frac{\partial \mathbf{r}_i}{\partial t} = 0$ (the coord trans does not dep on time explicitly)

⇒ $h = E$ (total energy)!

Energy Conservation and Time Invariance

$$h\left(q_j,\dot{q}_j,t\right)=\sum_j \dot{q}_j\frac{\partial L}{\partial \dot{q}_j}-L$$

is call the Jacobi Integral or the energy function.

h is closely related to the Hamiltonian function $H(q_i,p_i,t)$ that we will introduce later in the semester.

⇒ $h(q_i,\dot{q}_i,t)$ is a function of the generalized coordinates and their time derivatives

But, $H(q_i,p_i,t)$ is a function of the generalized coordinates and the associated generalized momenta